BETTER SOLUTION TO PAT'S PROBLEM

STEVE

Pat's two equations:

$$\frac{a(r^6-1)}{r-1} = A = 32327011.49 \cdot 6 \tag{1}$$

$$\frac{a(r^{18}-1)}{r-1} = B = 32327011.49 \cdot 6 + 51065497.13 \cdot 12 \tag{2}$$

Divide (2) by (1):

$$\frac{r^{18}-1}{r^6-1} = \frac{B}{A} \tag{3}$$

Simplify the left-hand side:

$$1 + r^6 + r^{12} = \frac{B}{A} \tag{4}$$

Let $x = r^6$:

$$1 + x + x^2 = \frac{B}{A} \tag{5}$$

Rewrite slightly:

$$x^2 + x + \left(1 - \frac{B}{A}\right) = 0 \tag{6}$$

Solve (6) using the quadratic formula:

$$x = \frac{-1 \pm \sqrt{1 - 4\left(1 - \frac{B}{A}\right)}}{2} \tag{7}$$

Use a calculator to find $\frac{B}{A} = 4.159308255$. Substitute into (7):

$$x = \frac{-1 \pm \sqrt{1 - 4\left(1 - 4.159308255\right)}}{2} = \frac{-1 \pm \sqrt{13.637233020}}{2} \tag{8}$$

Use the positive square root so we don't get a negative number:

$$x = \frac{-1 + \sqrt{13.637233020}}{2} = 1.346431221 \tag{9}$$

Date: March 20, 2014.

Since $r^6 = x$,

$$r = (1.346431221)^{1/6} = 1.050825723 \tag{10}$$

Thus the growth rate is 5% as before, and from (1),

$$a = A\frac{r-1}{r^6-1} = 32327011.49 \cdot 6 \cdot \frac{1.050825723 - 1}{1.050825723^6 - 1} = 28456622.66. \tag{11}$$

Thus first month's take is \$28,456,622.66.

This method explains why Maple found exactly 12 answers including complex ones. Equation (8) gives two values for x. Since $r^6 = x$ and every nonzero number has exactly six sixth roots over the complex numbers, there are twelve possible values for r.